

Systematic Listing

Model drawing (Whole numbers and Fractions) Guess & Check



27 March 2021

# **OUTLINE OF WORKSHOP:**

- Sharing on STAR strategy
- Sharing on Systematic Listing
- Hands-on Practice
- Sharing on Model Drawing
- Hands-on Practice
- Sharing on Guess & Check
- Hands-on Practice
- Parents'/Guardians' Involvement



# **OBJECTIVES**.

Able to use STAR strategy in solving problem sums

Able to use the different Heuristics (Systematic Listing, Model Drawing and Guess & Check) to solve Math problems



# STAR APPROACH

### <u>Study the problem.</u>

- -What is the important information given?
- -How can I connect, link or organise the information?

### Think of a plan.

- -What is the missing information I need to find?
- -What strategies should I use to find the missing information?



# STAR APPROACH

### Act on the plan.

- -How do I solve the problem?
- -Does each step makes sense?

### $\underline{\mathbf{R}}$ eflect on solution.

-How do I know if I had answered the problem?





# SYSTEMATIC LISTING

### **Purpose of systematic listing:**

- Come up with a list of the possible results of a given task by increasing or decreasing in number from the previous solution
- 2) Count the different ways of listing or to find the answer given the clues



### WORKED EXAMPLE: SYSTEMATIC LISTING

Q1.Peter wants to number his racing car collection.

<u>Each car must have a 2-digit number.</u>

He can <u>only use the digits 3, 5 and 8</u>.

How many cars can he number if each digit can be used more than once?

#### Step 1: Study the problem:

What is the important information given?

- Each car must have a 2-digit number
- Can only use the digits 3, 5 and 8
- Each digit can be used more than once

Step 1: <u>Study the problem</u>:

How can I link the information?

- I can only use digits 3, 5 and 8 to number each car with a 2digit number
- Each digit can be used more than once

### WORKED EXAMPLE: SYSTEMATIC LISTING

Q1.Peter wants to number his racing car collection.

<u>Each car must have a 2-digit number.</u>

He can <u>only use the digits 3, 5 and 8</u>.

How many cars can he number if each digit can be used more than once?

#### **Step 2: <u>Think of a plan:</u>** <u>What is the missing</u> information?

Number of cars that
 Peter can number if
 each digit can be used
 more than once

#### Step 2: Think of a plan:

<u>What strategy should I use? Why?</u>

- make a systematic list to show the possible 2-digit numbers starting with 3, then with 5 and lastly 8
- I can have a clearer view of the information given and I will not miss out any of the important details.

### WORKED EXAMPLE: SYSTEMATIC LISTING

#### Step 3: <u>A</u>ct on the plan:

How should I solve the problem?

Solve the problem by drawing a table to list the 2-digit numbers starting with 3, 5 or 8.

2-digit number starting	2-digit number starting	2-digit number starting	
with 3 (using digits 3, 5	with 5 (using digits 3, 5	with 8 (using digits 3, 5	
and/ or 8)	and/ or 8)	and/ or 8)	
33	53	83	
35	55	85	
38	58	88	
He can number 9 cars.			

#### Step 4: <u>R</u>eflect on the Solution:

How do I know if I have answered the problem?

- The question asked for the number of cars that can be numbered with 2-digit number using digits 3,5 and 8. My answer is 9.
- I did not miss out any possible numbers.

Step 3: <u>A</u>ct on the plan:

How do I know whether each step makes sense?

- All my numbers are 2-digit numbers.
- I use only the numbers 3, 5 and 8.
- I did not miss out any possible numbers.

### GUIDED EXAMPLE: SYSTEMATIC LISTING

Q2. Tom has a <u>5-cent stamp</u>, a <u>10-cent stamp</u> and a <u>20-cent stamp</u>. How many <u>different values of postage</u> can he form with the stamps?

**Step 1: <u>S</u>tudy the problem:** What is the important given information?

• a 5-cent stamp, a 10-cent stamp and a 20-cent stamp are used to form different values of postage Step 1: Study the problem:

How can I link the information?

• *Find the different values* he can form with the stamps.



### GUIDED EXAMPLE: SYSTEMATIC LISTING

Q2. Tom has a <u>5-cent stamp</u>, a <u>10-cent stamp</u> and a <u>20-cent stamp</u>. How many <u>different values of postage</u> can he form with the stamps?

**Step 2: <u>Think of a plan:</u>** What is the missing

information I need to find?

 Different values of postage Tom can be formed using the stamps

#### Step 2: Think of a plan:

<u>What strategy should I use to find the</u> <u>missing information? Why?</u>

• Make a systematic list to show the possible values of postage he can form with the stamps so that I can have a clearer view of the information and I will not miss out any of the important details.

### **GUIDED EXAMPLE: SYSTEMATIC LISTING**

#### Step 3: <u>Act on the plan:</u>

How do I solve the problem?

Make a systematic list to show the possible values of postage he can form 

	Values formed with 1	Values formed with 2		med with 2	Values formed with 3	
	stamp	stamps		mps	stamps	
	5 ¢	$5 \phi + 10 \phi = 15 \phi$		$D \phi = 15 \phi$	$5\phi + 10\phi + 20\phi = 35\phi$	
	10 ¢	5c + 20c = 25c		$0\phi = 25\phi$		
	20 ¢	$10\phi + 20\phi = 30\phi$		$30\phi = 30\phi$		
	He can form <u>7</u> different values of postage.			Step 3: <u>A</u> ct on the <u>How do I know w</u> • <i>I used 3 stamps</i>	e <b>plan:</b> hether each step makes sense? for the possible values.	
<ul> <li>Step 4: <u>R</u>eflect on the Solution:</li> <li><u>How do I know if I have answered the problem?</u></li> <li>The question asked for the different values of postage using 5 cents, 10 cents and 20 cents stamps.</li> </ul>		<ul> <li>I added the value</li> <li>I did not miss ou</li> <li>I do not need to because I am aske</li> </ul>	tes correctly. It any stamps. include the unit '¢' in my final answe ed to find the different values.	r		
<ul> <li>I did not miss out any possible values.</li> </ul>						

# Now Let Us Try

P1. A towel printing shop uses only the numbers 1, 4, 7 and 9 to print 2-digit numbers on its towels. Each digit cannot be repeated in the number.How many possible 2-digit numbers can the shopkeeper use to print her towels?

2-digit numbers starting with 1	2-digit numbers starting with 4	2-digit numbers starting with 7	2-digit numbers starting with 9
14	41	71	91
17	47	74	94
19	49	79	97

She can use <u>12</u> possible 2-digit numbers to print her towels.





# MODEL DRAWING

### **Purpose of model drawing:**

- Provides a visual way of picturing a problem
- 2) Simplify a problem to a more obvious solution



### WORKED EXAMPLE: MODEL DRAWING

Q1. Jessie has 2300 stamps.

Her friend gives her 1470 stamps and her sister gives her another 680 stamps. How many stamps does Jessie have now?

#### **Step 1: <u>S</u>tudy the problem:** What is the important given information?

- Jessie has 2300 stamps
- Her friend gives her 1470 stamps
- *Her sister gives her another 680 stamps*

#### **Step 1: <u>Study the problem:</u>** <u>How can I link the information?</u>

• I have to add on the stamps that were given to Jessie.



### Worked Example: Model Drawing

Jessie has 2300 stamps. Her friend gives her 1470 stamps and her sister gives her another 680 stamps. How many stamps does Jessie have now?

### **Step 2: <u>Think of a plan:</u>** <u>What is the missing</u> <u>information?</u>

• I need to find the total number of stamps Jessie has in the end.

#### Step 2: <u>T</u>hink of a plan:

What strategy should I use? Why?

 I can draw a part-whole model to find the total number of stamps Jessie has.
 By drawing the model, it gives me a clearer picture of the number of stamps that she has.

### WORKED EXAMPLE: MODEL DRAWING

#### Step 3: <u>A</u>ct on the plan:

How should I solve the problem?

Draw a **<u>part-whole model</u>** to find the total number of stamps Jessie has.



2300 + 1470 = 37703770 + 680 = 4450

### Jessie has 4450 stamps now.

Step 4: <u>Reflect on the Solution:</u>

How do I know if I have answered the problem?

- The question asked for the number of stamps Jessie has now and my answer is 4450.
- I can check by working backwards:

4450 - 680 = 3770 and 3770 - 1470 = 2300

### GUIDED EXAMPLE: MODEL DRAWING

Q2. There are 1005 girls in a school. The number of boys is 340 less than the number of girls. What is the total number of pupils in the school?

**Step 1: <u>S</u>tudy the problem:** What is the important given information?

- 1005 girls
- *number of boys is 340 less than the number of girls*

Step 1: Study the problem:

How can I link the information?

- *Number of girls* = <u>1005</u>
- There are <u>340 more</u> girls than boys.



### GUIDED EXAMPLE: MODEL DRAWING

Q2. There are 1005 girls in a school. The number of boys is 340 less than the number of girls. What is the total number of pupils in the school?

Step 2: Think of a plan: What is the missing information I need to find? • I need to find the number of boys and the total number of pupils.

#### Step 2: Think of a plan:

<u>What strategy should I use to find the missing</u> <u>information? Why?</u>

- Draw a <u>comparison model</u> to find the number of girls before finding the total number of pupils in the school.
- This is drawn to have a clearer picture of the comparison between the number of boys and the number of girls.

### GUIDED EXAMPLE: MODEL DRAWING

#### Step 3: <u>A</u>ct on the plan:

How do I solve the problem?

 Draw a comparison model to find the number of girls and total number of pupils in school
 Step 3: Act on the plan:



Number of boys = 1005 - 340 = 665Total number of pupils =1005 + 665 = 1670

There are 1670 pupils in the school.

Step 3: <u>A</u>ct on the plan: <u>How do I know whether each step makes</u> <u>sense?</u>

• My steps make sense because <u>since there</u> <u>are less boys, the number of boys should</u> <u>be less than the number of girls</u>.

**Step 4: <u>R</u>eflect on the Solution:** How do I know if I have answered the problem?

- The question asked for the total number of pupils and my answer is <u>1670</u>.
- I can check by working backwards.
- 1670 665 (boys) = 1005 (girls)

### Now Let Us Try

P1. The number of pages in a dictionary is 1845.A storybook has 1380 less pages than a dictionary.Find the total number of pages in the two books.



Number of pages in a storybook  $\rightarrow$  1845 – 1380 = 465

Total number of pages in a dictionary and a storybook  $\rightarrow$  1845 + 465 = 2310

There are **2310** pages in the two books.





### **GUESS AND CHECK**

### **Purpose of Guess and Check :**

- 1) To make a calculated guess
- 2) Simplify a problem to a more obvious solution



#### Q1. There are <u>fifteen 2-mark and 3-mark questions</u> in a Mathematics test. If the <u>total marks of the test is 35 marks</u>, <u>how many questions of each</u> <u>kind are in the test</u>?

#### Step 1: Study the problem: What do I need to find?

• Number of 2-mark and 3-mark questions in the test

# What are the important information given?

- 15 questions in a test consisting 2mark and 3-mark questions
- total marks is 35

**Step 1: <u>S</u>tudy the problem:** Do you know the number of 2-mark questions or number of 3-mark questions?

• I know that the total number of 2mark questions and 3-mark questions is 15



#### Q1. There are <u>fifteen 2-mark and 3-mark questions</u> in a Mathematics test. If the <u>total marks of the test is 35 marks</u>, <u>how many questions of each</u> <u>kind are in the test</u>?

#### Step 2: <u>T</u>hink of a plan:

Do I have all the information to solve the problems?

- I know that there are 2-mark and 3-mark questions in the test
- There are 15 questions altogether
- The total mark is 35

**Step 2: <u>T</u>hink of a plan:** What are the possible headings for the guess and check table?

- Draw a table with 6 columns
- $l^{st}$  column  $\rightarrow$  number of 2-mark questions
- $2^{nd}$  column  $\rightarrow$  total marks for 2-mark questions
- $3^{rd}$  column  $\rightarrow$  number of 3-mark questions
- $4^{th}$  column  $\rightarrow$  total marks for 3-mark questions
- $5^{th}$  column  $\rightarrow$  sum of all the marks
- $6^{th}$  column  $\rightarrow$  check if the sum of marks matches 35.



Q1. There are <u>fifteen 2-mark and 3-mark questions</u> in a Mathematics test. If the <u>total marks of the test is 35 marks</u>, <u>how many questions of each</u> <u>kind are in the test</u>?

#### Step 2: Think of a plan:

What strategy should I use to solve the problem?

- Make logical guesses on the number of 2-mark and 3-mark questions and check if the sum of their marks matches the total marks for the test.
- *Keep guessing and checking until you get the correct answer.*



#### Step 3: <u>A</u>ct on the plan:

How should I solve the problem?

Solve the problem by listing the combinations of 2-mark and 3-mark questions that make up 15 questions

No. of 2- mark questions	Total marks for 2-mark questions	No. of 3- mark questions		Total marks for 3-mark questions	Sum of marks	Check Does it match 35 marks?
7	$7 \ge 2 = 14$	8		8 x 3 = 24	14 + 24 = 38	×(Sum is too big)
12	$12 \ge 2 = 24$	3		$3 \ge 3 = 9$	24 + 9 = 33	×(Sum is too small)
10	$10 \ge 2 = 20$	5		$5 \ge 3 = 15$	20 + 15 = 35	$\checkmark$
· · · ·				4: <u>R</u> eflect on the	Solution:	
There are 10 of the 2-mark questions and 5 of the 3-mark questions.			Have • Th ma ma	e I answered the e question aske rk questions an rk questions an	e question? ed for the numb d my answer is d 5 of the 3-ma	oer of 2-mark and 3- there are 10 of the a ork questions.

### GUIDED EXAMPLE: GUESS AND CHECK

#### Q2. A carpark has 20 motorcycles and cars. There are 64 wheels altogether. How many motorcycles does the carpark have?

#### Step 1: <u>Study</u> the problem:

What do I need to find?

- Number of motorcycles in the carpark What are the important information given?
- Total number of motorcycles and cars = <u>20</u>
- Total number of wheels = <u>64</u>
- A car has <u>4</u> wheels and a motorcycle has <u>2</u> wheels.

#### Step 1: Study the problem:

Do you know the number of motorcycles or the number of cars in the carpark?

• I know that the total number of both motorcycles and cars is <u>20</u>.



### GUIDED EXAMPLE: GUESS AND CHECK

#### Q2. A carpark has 20 motorcycles and cars. There are 64 wheels altogether. How many motorcycles does the carpark have?

#### Step 2: <u>T</u>hink of a plan:

What is the missing

information I need to find?

- <u>The total number of</u> <u>motorcycles and cars is 20</u> <u>and the total number of</u> <u>wheels is 64.</u>
- <u>A motorcycle has 2 wheels</u> and a car has 4 wheels.
- I have to find the number of motorcycles and the number of cars.

#### Step 2: Think of a plan:

What are the possible headings for the guess and check table?

- Draw table with 6 columns
- $l^{st}$  column  $\rightarrow$  number of motorcycles
- $2^{nd}$  column  $\rightarrow$  total number of wheels of motorcycles
- 3<sup>rd</sup> column → number of cars
- 4<sup>th</sup> column → total number of wheels of cars
- $5^{th}$  column  $\rightarrow$  sum of all the wheels
- $6^{th}$  column  $\rightarrow$  check if the sum of wheels matches 64

#### Q2. A carpark has 20 motorcycles and cars. There are 64 wheels altogether. How many motorcycles does the carpark have?

#### Step 2: Think of a plan:

What strategy should I use to solve the problem?

- Make <u>logical guesses</u> on the number of <u>motorcycles</u> and <u>cars</u> and check if the sum of their <u>wheels</u> matches the total number of wheels.
- Keep guessing and checking until you get the correct answer.



### GUIDED EXAMPLE: GUESS AND CHECK

### Step 3: <u>A</u>ct on the plan:

How do I solve the problem?

 Make a list to show the possible combinations of motorcycles and cars that make up 20

	No of motorcycles	No of wheels of motorcycles	No of ca	ns No of wheels of cars	Sum of wheels	Check Does it match 64 wheels?
	10	10 X 2 = 20	10	10 X 4 = 40	20 + 40 = 60	×(Sum is too small)
	12	$12 \ge 2 = 24$	8	8 x 4 = 32	24 + 32 = 56	×(Sum is too small)
	8	8 x 2 = 16	12	$12 \times 4 = 48$	16 + 48 = 64	$\checkmark$
There are <u>8</u> motorcycles and <u>12</u> cars.			Step How • Th	<b>4:</b> <u><b>Reflect on the Second S</b></u>	olution: answered the <u>p</u> r the number of	oroblem? motorcycles and

# Now Let Us Try

P1. There are a total of 30 chickens and cows in a farm. If there are 82 legs altogether, how many animals of each type are there?

No. of chickens	No. of legs of chickens (2 legs each)	No. of cows	No. of legs of cows (4 legs each)	Total no. of legs	Check: Does it match 82?
15	$15 \ge 2 = 30$	30 - 15 = 15	$15 \ge 4 = 60$	30 + 60 = 90	× (Total is too big)
20	$20 \ge 2 = 40$	30 - 20 = 10	$10 \ge 4 = 40$	40 + 40 = 80	× (Total is too small)
19	19 x 2= 38	30 – 19 = 11	$11 \ge 4 = 44$	38 + 44 = 82	$\checkmark$

There are **19 chickens** and **11 cows**.



### ADDITIONAL QUESTIONS TO TRY...

Mdm Lim runs an ice-cream stall. She has waffles, cups and ice-cream cones. She offers 3 flavours of ice-cream: vanilla, chocolate and strawberry. How many different combinations can she offer her customers if they can choose one flavour of ice-cream on waffle or cup or cone? (Use systematic listing strategy)

Туре	Ice-cream flavour
Waffle	Vanilla
	Chocolate
	Strawberry
Cup	Vanilla
	Chocolate
	Strawberry
Cone	Vanilla
	Chocolate
	Strawberry

Waffle	Cup	Cone
Vanilla	Vanilla	Vanilla
Chocolate	Chocolate	Chocolate
Strawberry	Strawberry	Strawberry

#### She can offer $\underline{9}$ different combinations.



## ADDITIONAL QUESTIONS TO TRY...

There were 5080 passengers on a cruise ship.

1325 of the passengers were children.

How many more adults than children were there on the ship?

(Use model drawing strategy)





Number of Adults  $\rightarrow 5080 - 1325$ = 3755 Difference in the number of adults and children  $\rightarrow 3755 - 1325$ = 2430 There are **2430** more adults.



## ADDITIONAL QUESTIONS TO TRY...

A pen costs \$3 each and a file costs \$5 each. Mrs Lim bought a total of 15 pens and files for \$57. How many items of each kind did she buy? (Use guess and check strategy)



No. of pens	Cost of pens (\$3 each)	No. of files	Cost of files (\$5 each)	Total cost of pens and files	Check Does it match \$57?
7	7 x \$3 = \$21	15 - 7 = 8	$8 \ge $5 = $40$	\$21 + \$40 = \$61	× (Total is too big)
10	10 x \$3 = \$30	15 - 10 = 5	5 x \$5 = \$25	\$30 + \$25 = \$55	× (Total is too small)
9	9 x \$3 = \$27	15 - 9 = 6	6 x \$5 = \$30	\$27 + \$30= \$57	$\checkmark$

She bought <u>9 pens</u> and <u>6 files</u>.



# SUMMARY

STAR Strategy
Systematic listing
Model drawing
Guess & Check



# PARENTS'/GUARDIANS' INVOLVEMENT

- ATM –basic arithmetic weekly
- Math Around Us
- Mathemagician
- SLS
- Math Journal
- Communication and Reasoning
- Motivation & Support
- Be Present
- Making Connections
- Online Resources





# ATM – BASIC ARITHMETIC

- ATM will be sent home weekly
- Pupils to attempt the set of questions.
- Minimum 1 set per weekend
- Parents'/Guardians' endorsement is needed



# MATH IS AROUND US

- Show children that Math is part of our lives
- Relate whenever possible Math concepts to things we see around us and in our house so children can see that Math is relevant.
- Plan a virtual holiday/shopping for groceries

/baking for example where children calculate distance, budget, measure etc



# MATHEMAGICIAN

- One pupil will be the 'Mathemagician of the Week'
- Mathemagician can bring any of the following items to share with their classmates:
- Interesting Math articles
- Math games
- Math history
- Math facts
- Math storybooks





# STUDENT LEARNING SPACE (SLS)

- Weekly assignment
- Self-directed learning through exploration of Math lesson packages





# MATH JOURNAL

 Pupils will write down their reflection about their learning and understanding of Math concepts:

Example:

- Activity-based lessons
- Math addition/subtraction/multiplication/division facts
- Concepts they have learnt
- Strengths / Areas for improvement
- Weekly journaling



# **MOTIVATION & SUPPORT**

- Affirm your child's effort in learning Mathematics
- Share personal experiences dealing with failures
- Encourage your child to persevere and be self-directed in their learning (E.g: Exploring the various lesson packages in SLS, reading Math storybooks, watch Math videos, read Math articles in magazines or newspapers)





# **COMMUNICATION & REASONING**

- Now it's our turn to ask why
- Let the children explain
- Make their thinking visible
- Correct misconceptions
- Build confidence through oracy/ critical thinking skills



# **BE PRESENT**

#### Be Present

- Show interest in their learning through regular chats with them
- Let your child verbalise their solutions to selected questions
- Encourage alternative possible solutions
- Learn together with your child



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# MAKING CONNECTIONS

#### Making connections

- Seize opportunities to relate Math to the real world

Examples:

- Shopping  $\rightarrow$  adding and counting money, choosing value for money products
- Car park rate
- Bill payments
- Telling time
- Measurement of ingredients used in cooking
- Fractions in food
- Shapes and patterns in buildings





# **USEFUL WEBSITES**

- Online Math resources and questions: <u>https://sg.ixl.com/math/</u>
- Math Songs:
- Numberock Youtube Channel
- Math Antics Youtube Channel
- Math games:
- <u>https://www.coolmathgames.com/l-number-games</u>
- <u>https://www.mathplayground.com/</u>
- Math tools and practices:
- <u>https://www.matholia.com/sg/apps/tools?grade\_id=3</u>





